A NEW APPROACH FOR SOLVING ANTI-JAMMING GAMES IN STOCHASTIC SCENARIOS AS PURSUIT-EVASION GAMES

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ABSTRACT

We solve a communication problem between a UAV and a set of relays, in the presence of a jamming UAV, using differential game theory tools. The standard solution involves a set of coupled Bellman equations which are hard to solve. We propose a new approach in which this kind of games can be approximated as pursuit-evasion games. The problem is posed in terms of optimizing capacity and it is approximated as a zero-sum, pursuit-evasion game. This game is solved using a set of differential equations known as Isaacs equations and simulations are run in order to validate the results.

Index Terms— Pursuit-evasion games, Isaacs equations, mobile networks, UAVs

1. INTRODUCTION

In the last years, a large research has been done related to Unmanned Aerial Vehicles (UAVs), either in military or civil scenarios. In a formation, communication between vehicles must be wireless. Thus these links are vulnerable to jamming attacks. This is an area of research where different attack / defense strategies have been proposed. A wide variety of techniques are used, as spectral channel surfing and spatial positioning of the nodes [1], game theory tools [2], [3], [4], [5] or the use of a honey-pot node [6]. A general survey of jamming techniques is presented in [7].

If the jammer and communicating nodes are mobile, the attack can be modeled as a zero-sum, non-cooperative, differential game [8]. There are several tools dedicated to analyze this kind of games, especially for two player games [9], [10], [11], [12]. There are specific solutions for some multi-player games, such as [13], [14], [15], [16], [17]. The main tools used are the Hamilton-Jacobi-Bellman-Isaacs equations, which are difficult to solve to obtain an analytical solution. In some specific games, the game can be solved using only Isaacs equations [10], which greatly simplifies the analysis.

In this paper, the case in which there is one UAV trying to communicate with relay nodes while another UAV tries to jam the communications is modeled using differential game theory. The main contribution of the paper is posing the problem in terms of optimizing capacity and under some hypotheses, approximating it as a pursuit-evasion game using Isaacs' tools. These tools traditionally are used in physical pursuit-evasion settings: here, we extend them to communications. To the best of our knowledge, this has not been done yet and allows obtaining a novel approach in which communications related problems can be solved using well known pursuit-evasion game tools. We exemplify this new approach with the jamming game proposed.

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In Section 2, we present tools for differential games, with special emphasis in the situations where Isaacs equations apply. In Section 3, the problem described above is detailed and posed. In Section 4, we propose a specific dynamic setting and obtain the trajectory equations for both UAVs. Finally, in Section 5, we run some simulations in order to validate our approach.

2. DIFFERENTIAL GAMES

In a two player, zero-sum, pursuit evasion game, there are two players called pursuer and evader, with controls $u_1(t)$ and $u_2(t)$ respectively and state vector $\mathbf{x}(t)$. Since it is a zero-sum game, both players have the same cost functional with opposite sign, where one player tries to maximize the payoff function and the other tries to minimize it. This payoff function is:

$$\pi\left(\mathbf{x}(t), u_1(t), u_2(t)\right) = \int_0^{t_f} L\left(\mathbf{x}(t), u_1(t), u_2(t)\right) dt + G\left(\mathbf{x}(t_f)\right) \tag{1}$$

In a pursuit-evasion game, final and running cost are $G = 0$ and $L = 1$, respectively; and thus, $\pi = t_f$. Therefore, players try to minimize or maximize the capture time (t_f).

The game outcome obtained if both players implement their optimal strategy will be the value function $V(\mathbf{x})$, for any state $\mathbf{x}$ in the state space. The gradient of the value function is denoted as ∇V. Finally, the concrete setup of the system will provide the dynamic equation, which will be expressed in the following form: $\dot{\mathbf{x}} = f(\mathbf{x}, u_1, u_2)$. This game will present the next Hamiltonian, which is the key element of the solution procedure:

$$\begin{aligned} H(\mathbf{x}, \nabla V, u_1, u_2) &= \nabla V \cdot f(\mathbf{x}, u_1, u_2) + L(\mathbf{x}, u_1, u_2) \\ &= \nabla V \cdot f(\mathbf{x}, u_1, u_2) + 1 \end{aligned} \tag{2}$$

2.1. Bellman and Isaacs approaches

Bellman and Isaacs equations are two methods for solving differential games. The former allows us to solve games using feedback information structures, at the cost of solving a partial differential equation. The latter allows us to solve open loop games, where ordinary differential equations are to be solved [18]. Let us start from Hamilton - Jacobi - Bellman equation (HJB), which states that:

$$H^* + \frac{\partial V}{\partial t} = 0 \tag{3}$$

Isaacs' main equation [10, p. 67] can be seen as a particular case, when $\frac{\partial V}{\partial t} = 0$ and, hence, $H^* = 0$. Also, the following additional conditions must be satisfied:

- The game is two players, zero-sum, pursuit-evasion type. Being a pursuit evasion game implies that final time is free (i.e., to be optimized), but this condition can be relaxed [10, p. 34].

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- The game must be deterministic.
- The Hamiltonian is separable on its controls [10, p. 35].

Thus, if V, the game value cost function, does not depend explicitly on time, and these conditions are satisfied, Isaacs approach becomes a particularization of Bellman equation. This condition is also satisfied, according to [19, p. 36], when the optimal control problem that is being solved is time-invariant and the final time is free, i.e., needs to be optimized.

3. PROBLEM DESCRIPTION

In this section, we pose a capacity game and approximate it as a pursuit-evasion one. Let us suppose that there are two UAVs and a high number of relays, which can be static or dynamic. The communicator tries to communicate with the relays, whereas the jammer tries to jam this communication. Thus, both players have opposite objectives and, hence, a zero-sum game between them is posed.

The total capacity in this scenario can be computed as the sum of the different capacities at each relay. Considering a free space propagation model, orthogonal modulation and using Shannon's capacity formula, the total capacity per bandwidth unit of the system depends on the SINR as follows [20]:

$$C_t = \sum_{i=1}^{N} \log_2(1+\mathrm{SINR}_i) = \sum_{i=1}^{N} \log_2\left(1 + \frac{\frac{P_c}{d^2_{c,ri}}}{N_0 + \frac{P_j}{d^2_{j,ri}}}\right) \quad (4)$$

In the expression before, P_c and P_j are the communicator and the jammer transmission fixed power, respectively; $d_{c,ri}$ and $d_{j,ri}$ are the euclidean distances between the communicator or the jammer and relay i, respectively, considering that there are N relays; and N_0 is the noise floor power. In order to optimize the expression above, it would be necessary to know the position of each relay in every time instant (and their dynamics if they were mobile).

If there is no knowledge about relays positions, a different approach is required. Let us suppose that relays and UAVs move in the $\mathbb{R}^3$ Cartesian space, thus, in every time instant, the position is defined by the vector (x, y, z). Let us assume that both UAVs move on the same plane (i.e., they have constant z-coordinate), and that all mobile relays also move on the same plane. Let us define ϵ as the distance between the plane of relays and the UAVs plane. Assuming that the jammer power at the receiver is much bigger than noise, the SINR can be approached by the SIR. If the relays positions in the plane are considered to be a random vector $S = (S_x, S_y)$, with arbitrary probability density function $f_i(S_{x,i}, S_{y,i})$, the game payoff can be computed as the mathematical expectation of the SIR as follows:

$$\mathbb{E}\{C_t(S_x, S_y)\} = \int\int \sum_{i=1}^{N} \log_2\left(1 + \frac{P_c}{P_j}\cdot\frac{d^2_{j,ri}(S)}{d^2_{c,ri}(S)}\right) f_i(S_{x,i}, S_{y,i}) dS_i \quad (5)$$

where $dS_i = dS_{y,i}dS_{x,i}$, and $d^2_{c,ri}(S) = (x_c - S_{x,i})^2 + (y_c - S_{y,i})^2 + \epsilon^2$ and $d^2_{j,ri}(S) = (x_j - S_{x,i})^2 + (y_j - S_{y,i})^2 + \epsilon^2$ are, respectively, the distance between the communicator or the jammer and relay i, whose plane-coordinates are $(S_{x,i}, S_{y,i})$. If the random variables S_i are considered to be independent and identically distributed (i.i.d.) and assuming that relays follow a uniform distribution in the interval $[-D, D]$ in coordinates X and Y, the expression in (5) becomes:

$$\mathbb{E}\{C_t(S_x, S_y)\} = \int_{-D}^{D}\int_{-D}^{D} N\cdot\log_2\left(1+\frac{P_c}{P_j}\cdot\frac{d^2_{j,r}(S)}{d^2_{c,r}(S)}\right)\frac{1}{4\cdot D^2}dS \quad (6)$$

where dS_i becomes $dS = dS_y dS_x$, and $d^2_{c,ri}(S)$ and $d^2_{j,ri}(S)$ become $d^2_{c,r}(S) = (x_c - S_x)^2 + (y_c - S_y)^2 + \epsilon^2$ and $d^2_{j,r}(S) = (x_j - S_x)^2 + (y_j - S_y)^2 + \epsilon^2$, respectively. The expression in (6) is hard to solve analytically, hence, we will use an approximation. Integrating (6) with respect to S_x and simplifying the results, considering that the jammer and evader are far from the region borders and that ϵ is small compared to the region size, which means that $D \gg |x_c|, D \gg |x_j|, D \gg \epsilon$, we can simplify and obtain:

$$\mathbb{E}\{C_t(S_x|S_y)\} = \int_{-D}^{D} N\cdot\log_2\left(1+\frac{P_c}{P_j}\cdot\frac{d^2_{j,r}(S_x,S_y)}{d^2_{c,r}(S_x,S_y)}\right)\frac{1}{2D}dS_x$$
$$\approx N\cdot\left(\log_2\left(1+\frac{P_c}{P_j}\right) - \frac{\sqrt{(y_c-s_y)^2}\cdot\pi}{D\cdot\log(2)} + \frac{\pi\sqrt{\left(\frac{P_j}{P_c}+1\right)\left((y_j-s_y)^2+\frac{P_j}{P_c}(y_c-s_y)^2\right)+\frac{P_j}{P_c}(x_c-x_j)^2}}{D\left(\frac{P_j}{P_c}+1\right)\log(2)}\right) \quad (7)$$

If this approximation is integrated with respect to S_y and simplified using the same hypotheses than before but with respect to the Y coordinate ($D \gg |y_c|, D \gg |y_j|$), we can simplify the expression in (6) to:

$$\hat{\mathbb{E}}\{C_t(S_x, S_y)\} = N\cdot\left(\log_2\left(1+\frac{P_c}{P_j}\right) + \frac{\frac{P_j}{P_c}\cdot r\cdot \mathrm{arcsinh}\left(\frac{D\left(1+\frac{P_j}{P_c}\right)}{\sqrt{\frac{P_j}{P_c}\cdot r}}\right)}{2\cdot D^2\left(1+\frac{P_j}{P_c}\right)^2\log(2)}\right) \quad (8)$$

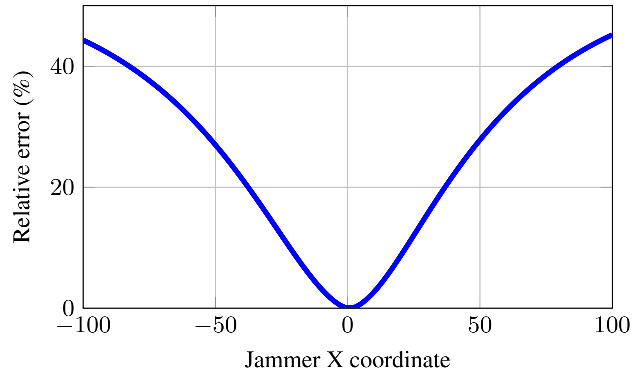


Fig. 1. Relative error between the approximated function $\hat{\mathbb{E}}\{C_t(S_x, S_y)\}$ and the average of 100 realizations $\mathbb{E}\{C_t(S_x, S_y)\}$. Communicator is fixed in $(x_c, y_c) = (2, 1)$ and the jammer is moved along main diagonal ($x_j = y_j$).The error is smaller when the hypotheses are satisfied and both UAVs are far from the edges ($|x_c|, |y_c| \ll D$ and $|x_j|, |y_j| \ll D$).

where $r = (y_c - y_j)^2 + (x_c - x_j)^2$. Hence, the capacity depends on r, the norm of the vector pointing from the communicator to the jammer: the bigger this norm is, the bigger capacity the system will have. Thus, the jammer wants to minimize capacity and that means trying to be close to the communicator, whereas the communicator tries to maximize capacity and that means being as far as possible from the jammer. This is also the idea behind pursuit-evasion games, hence, we will approximate the original capacity game with a pursuit-evasion one.

In order to estimate the performance of the capacity approximation in (8), a simulation has been run. Distributing $N = 100$ relays uniformly over a square of side $2D = 200$, the communicator has been placed on $(x_c, y_c) = (2, 1)$ and the jammer was placed along the main diagonal ($x_j = y_j$), with $\epsilon = 1$ and $P_c = P_j = 1$. We computed and averaged the empirical capacity for 100 realizations. The relative error between this average and the theoretical expression in (8) can be seen in Figure 1. The main conclusion we get is that once we can neglect border effects, this approximation is accurate.

4. PURSUIT-EVASION GAME OF TWO UAVS

In this section, the two-person, zero-sum, pursuit-evasion game that appears when approximating the problem described in Section 3 will be solved using Isaacs method, described in [10, ch. 4]. We consider each UAV to have a constant acceleration that will be F_p for the pursuer and F_e for the evader. A friction limit will be used, for the speed not to grow unbounded denoted by k_p and k_e for the pursuer and evader, respectively. Therefore, the maximum speed will be F/k. This setup is an extension to Isaacs "isotropic rocket" game [10, pp. 105-116], but considering that pursuer and evader have the same dynamics.

4.1. Dynamics of the UAVs

Each player control variable will be their heading angle with respect to Y-axis, which will be noted ϕ for the pursuer and ψ for the evader. There are 8 states: the position coordinates (x_p, y_p and x_e, y_e for pursuer and evader) and the velocities components (u_p, v_p and u_e, v_e for pursuer and evader), while in Isaacs problem there are 6 states [10, p. 106]. Hence, the dynamic equation is:

$$\begin{pmatrix} \dot{x_p} \\ \dot{y_p} \\ \dot{u_p} \\ \dot{v_p} \\ \dot{x_e} \\ \dot{y_e} \\ \dot{u_e} \\ \dot{v_e} \end{pmatrix} = \begin{pmatrix} u_p \\ v_p \\ F_p \cdot \sin(\phi) - k_p \cdot u_p \\ F_p \cdot \cos(\phi) - k_p \cdot v_p \\ u_e \\ v_e \\ F_e \cdot \sin(\psi) - k_e \cdot u_e \\ F_e \cdot \cos(\psi) - k_e \cdot v_e \end{pmatrix} \tag{9}$$

4.2. Control optimization

The Hamiltonian is built according to Isaacs "Main Equation" [10, p. 67]:

$$\max_{\psi} \min_{\phi} \sum_i V_{x_i} \cdot f_i + L = 0 \tag{10}$$

Since this is a pursuit-evasion game, the game outcome is the final time. Thus, the running and final cost are $L = 1$ and $G = 0$, respectively. Consider that $\rho_p = \sqrt{V_{u_p}^2 + V_{v_p}^2}$ and $\rho_e = \sqrt{V_{u_e}^2 + V_{v_e}^2}$. Using the dynamics in (9) and solving the maximization and minimization problems (taking advantage of the fact that controls are separable) yields the following solution for (10), whose left hand side will be noted as main equation (ME):

$$\begin{aligned} 1 + V_{x_p} \cdot u_p + V_{y_p} \cdot v_p - \rho_p \cdot F_p - k_p(V_{v_p} \cdot v_p + V_{u_p} \cdot u_p) \\ + V_{x_e} \cdot u_e + V_{y_e} \cdot v_e + \rho_e \cdot F_e - k_e(V_{v_e} \cdot v_e + V_{u_e} \cdot u_e) = 0 \end{aligned} \tag{11}$$

4.3. Retrogressive Path Equations

The next step in Isaacs' method is to obtain the Retrogressive Path Equations (RPE). There will be 16 RPE equations: one per state and another one per each component of the gradient of the value function, while in Isaacs original setup there were 12 [10, p. 107] . The 8 equations that depend on the dynamics equations are obtained from (9) using the variable change $\tau = t_f - t$. The other 8 RPE are obtained from the gradient of the value function, according to the following expression [10, p. 82]:

$$\mathring{V_k} = \frac{dV_k}{d\tau} = \sum_i V_i \cdot f_{i,k} + L_k = \frac{\partial \mathrm{ME}}{\partial x_k} \tag{12}$$

where $\mathring{V_k}$ is the derivative with respect to τ. These RPE are obtained through derivation of the ME with respect to each state variable.

4.4. Final conditions

In order to determine the final conditions, we must define the terminal surface (i.e., the surface where the pursuer captures the evader). By considering that the capture distance is l, the surface capture will be the ball whose center is the evader position: when the pursuer enters that ball, the game ends and capture occurs. Hence, the termination surface will be the sphere in which the distance between pursuer and evader equals l, the capture distance. It can be parametrized using $n - 1$ variables (where n is the number of states) as follows:

$$h = \begin{pmatrix} x_p \\ y_p \\ u_p \\ v_p \\ x_e \\ y_e \\ u_e \\ v_e \end{pmatrix} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_1 + l \cdot \sin(s_5) \\ s_2 + l \cdot \cos(s_5) \\ s_6 \\ s_7 \end{pmatrix} \tag{13}$$

Using (13), it is possible to obtain the final conditions with the following expression, taking into account that in this game, the final cost G is zero (Subsection 4.2):

$$\frac{\partial G}{\partial s_k} = \sum_i V_i \cdot \frac{\partial h}{\partial s_k} \tag{14}$$

We remark that these are final time conditions ($t = t_f$), but in retro time, they are initial conditions ($\tau = 0$), as it is implicit in the variable change done. Hence, the s_i are initial conditions in retro time, but final conditions in normal time.

4.5. RPE integration

We can obtain the optimal controls for both players solving the RPE equations (12), related to the gradient of the value function.This yields that optimal controls are constant and equal for both players, with the form $\phi^* = \psi^* = s_5$. The same solution is obtained for the original setup [10, p. 109].

By integrating the RPE equations related to state variables we obtain:

$$
\begin{aligned}
x_p &= s_1 + s_3 \cdot \frac{1 - e^{k_p \cdot \tau}}{k_p} + F_p \cdot \sin(s_5) \cdot \frac{e^{k_p \cdot \tau} - 1 - k_p \cdot \tau}{k_p^2} \\
u_p &= s_3 \cdot e^{k_p \cdot \tau} + F_p \cdot \sin(s_5) \cdot \frac{1 - e^{k_p \cdot \tau}}{k_p} \\
x_e &= s_1 + l \cdot \sin(s_5) + s_6 \cdot \frac{1 - e^{k_e \cdot \tau}}{k_e} + F_e \cdot \sin(s_5) \cdot \frac{e^{k_e \cdot \tau} - 1 - k_e \cdot \tau}{k_e^2} \\
u_e &= s_6 \cdot e^{k_e \cdot \tau} + F_e \cdot \sin(s_5) \cdot \frac{1 - e^{k_e \cdot \tau}}{k_e}
\end{aligned}
\tag{15}
$$

where y_p, v_p, y_e and v_e have similar expressions, but $\sin(s_5)$ is replaced by $\cos(s_5)$, s_1 by s_2, s_3 by s_4 and s_6 by s_7. The equations in (15) give the optimal trajectories for both players, depending on the parameters used to describe the terminal sphere and the retro time τ, which are unknown. Since initial conditions are known (i.e, initial positions and speeds of both players), it is possible to obtain these parameters by equaling the equations in (15) to the initial conditions and particularized to $t = 0$, that is, $\tau = t_f - t = t_f$.

This system, is nonlinear and trigonometric and may be hard to solve. To simplify its resolution, we apply the same procedure that Isaacs used [10, pp. 110-111]: the final time t_f is obtained from the initial conditions and game parameters by squaring and adding these two identities and by using that $\cos^2(\alpha) + \sin^2(\alpha) = 1$:

$$
\begin{aligned}
x_p - x_e - u_p \left(\frac{e^{-k_p \cdot \tau} - 1}{k_p} \right) + u_e \left(\frac{e^{-k_e \cdot \tau} - 1}{k_e} \right) &= \sin(s_5) \cdot Q(\tau) \\
y_p - y_e - v_p \left(\frac{e^{-k_p \cdot \tau} - 1}{k_p} \right) + v_e \left(\frac{e^{-k_e \cdot \tau} - 1}{k_e} \right) &= \cos(s_5) \cdot Q(\tau)
\end{aligned}
\tag{16}
$$

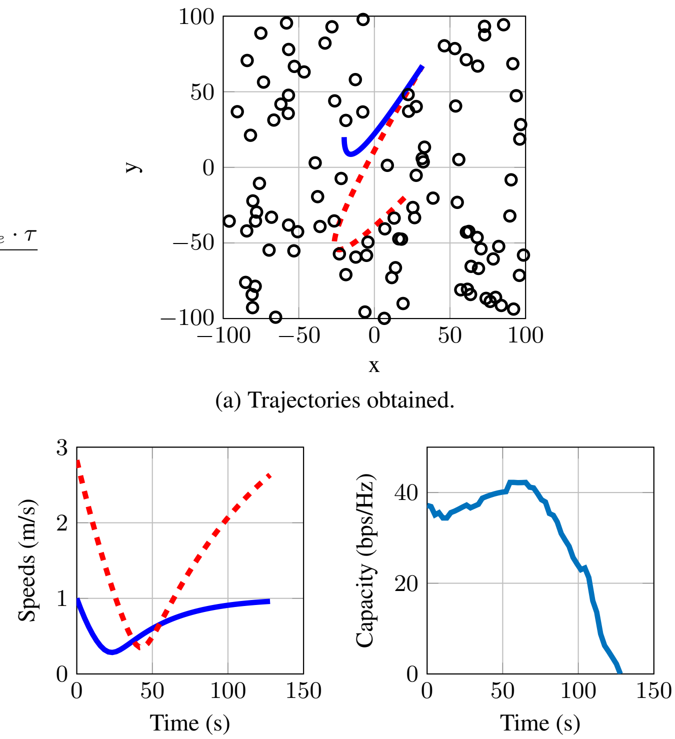
where

$$
Q(\tau) = \frac{F_e \cdot (e^{-k_e \cdot \tau} - 1 + k_e \cdot \tau)}{k_e^2} - l - \frac{F_p \cdot (e^{-k_p \cdot \tau} - 1 + k_p \cdot \tau)}{k_p^2}
\tag{17}
$$

Once that t_f has been obtained, it can be replaced in the system in (15). By particularizing for the initial time conditions and doing the following variable change: $w_1 = \cos(s_5)$, $w_2 = \sin(s_5)$, yields a linear system which can be solved using standard techniques (recall that $w_1^2 + w_2^2 = 1$).

5. SIMULATIONS

In order to validate the results presented above, a simulation has been run. The UAV parameters are $F_p = 0.05$, $k_p = 0.0125$ and, thus, $v_{max,p} = 4$; $F_e = 0.03$, $k_p = 0.03$ and hence, $v_{max,e} = 1$, $l = 0.1$. The relay region is a square, centered at the origin, whose side length is $2D = 200$. The relays are distributed randomly at each new simulation, following a uniform distribution in both X and Y coordinates. Initial conditions of both UAVs are randomly selected in each new simulation. A number of $N = 100$ static relays are considered, and $\epsilon = 1$. Considering that the jammer and the communicator are in the same spatial position, from (6), we can see that the SIR will be the same in all relay nodes, and equal to $\frac{P_c}{P_j}$. Provided that there is a minimum SINR threshold for communication to succeed and if noise is much smaller than interference, then SINR can be approximated by SIR and if the jammer knows the communicator power, it can adjust its own in order to cause all nodes to



(a) Trajectories obtained.

(b) Evolution of the speeds.

(c) Evolution of the system capacity.

Fig. 2. Example simulation results. Dashed line is pursuer, continuous is evader, points are relays. Jumps in the capacity function are due to the minimum SNR threshold: as game evolves, more relays have a SNR below that limit and when their SNR surpass that threshold, their contribution to total capacity becomes zero

fall below the SINR threshold and, hence, cause that total system capacity becomes zero. In order to satisfy this, the $SINR$ threshold considered is $SINR_{min} = 1$, with $P_c = 1$ and $P_j = 1.5$. A noise floor of power $N_0 = 10^{-4}$ is considered.

With these parameters, we simulated 100 UAVs optimal trajectories, using a time discretization of 50 points per trajectory. The capacity has been computed in each time step and the final results show that all simulations end up with capture and final capacity zero, so the jammer wins the game (as expected since the jammer is faster in this setup and its transmitted power is enough to cause all relay nodes SINR to fall below the threshold).

Figure (2) shows an example of trajectory on the plane, as well as the speed evolution of both players for initial conditions $(x_{p,0}, y_{p,0}) = (20, -20)$, $(x_{e,0}, y_{e,0}) = (-20, 20)$, $(u_{p,0}, v_{p,0}) = (-2, -2)$, $(u_{e,0}, v_{e,0}) = (0, -1)$.

6. CONCLUSIONS

We propose a new approach for solving games in stochastic scenarios, which consists in solving a pursuit-evasion game instead of a capacity one using an approximation. With this approach, a concrete application to an anti-jamming game has been studied and some simulations have been done in order to validate it. Our approach can be extended to other security games as network attacks.

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